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SLOW CYCLOTRON WAVE GROWTH BY PERIODIC INDUCTIVE STRUCTURES

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The Auto-Resonant Accelerator concept of collective ion acceleration is critically dependent for its success upon the availability of an effective means with which to grow the relevant slow cyclotron wave. We present a preliminary study of such growth via a two-dimensionally periodic slow wave structure. This structure consists of a z-slotted waveguide about which are placed conducting straps axially and azimuthally interrupted by capacitive gaps. Appropriate boundary conditions are derived without reference to concepts borrowed from low frequency circuit theory. These boundary conditions have been incorporated into a numerical code which performs linear normal mode analyses about self-consistently generated nonneutral relativistic electron beam equilibria. This same code may also be employed to examine the purely vacuum modes, which exhibit expected behavior. Questions of structure tuning are discussed. Initial results concerning wave growth are presented, and future activities indicated.

In the travelling-wave class of collective ion acceleration schemes, ions are placed in the trough of a large amplitude plasma wave that has been produced on a relativistic electron beam. The ion-wave system is then accelerated by increasing the phase-velocity of the wave through suitable spatial[1,2] or temporal[3] variation of system parameters. Crucial to the success of such schemes is the availability of an effective method with which to grow such large amplitude waves. For the Auto-Resonant Accelerator, where one is concerned with the slow cyclotron mode, several such methods have been investigated in the past. In one approach[4] explicit advantage is taken of the negative-energy nature of the slow cyclotron mode to grow the wave through the introduction of a dissipative element, such as a resistive liner. In another, perhaps more familiar method, growth is achieved by permitting the electron beam to interact with a slow-wave structure. The use of such structures is particularly attractive in this context, inasmuch as their spatial structuring offers at least the possibility of growing modes with prescribed desirable properties while discriminating against less favorable waves. In particular, the slow-wave structure consisting of a metallic helix surrounding the relativistic electron beam has hitherto been extensively investigated from this point of view.[4,5] In this paper, we present preliminary results of an investigation of another, quite different, slow-wave structure.

The system considered here consists of a z-slotted wave-guide around which are placed conducting straps. These straps are interrupted both axially and azimuthally by capacitive gaps to give rise to a two-dimensionally periodic structure. The entire system is enclosed within an outer cylindrical conducting wall. This structure, the resonant loop-drive, is depicted in Fig. 1. Interaction of an electron beam with such a structure may be viewed in two quite conceptually distinct, but physically equivalent ways. Firstly, the periodic structure may be considered to be an effective LC-circuit with the electron beam

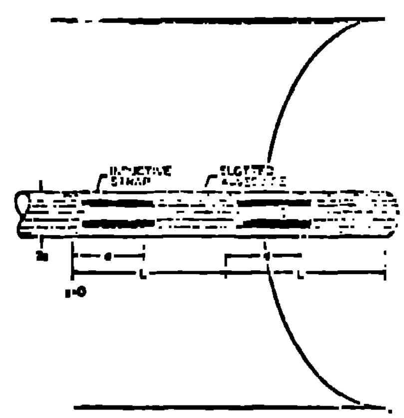

Fig. 1. Resonant loop drive slow wave structure.

serving as a source of electromotive force. The various geometric features of the slow-wave structure, such as the axial and azimuthal periodicities and corresponding strap lengths and widths, may then be adjusted to provide the effective LC-circuit with a resonant frequency appropriate to the beam mode whose growth is desired. Energy is consequently extracted from this mode; and since it is negative energy in character, the amplitude of the mode grows.

The second, perhaps more satisfying, way of viewing the interaction between the electron beam and the slow-wave structure being considered here is as being the solution of a boundary-value problem involving Maxwell's equations and a periodic boundary. In this view, the dispersion relation of the relevant vacuum mode mirrors in w-k space the periodicity in real space introduced by the boundary conditions. This mode is consequently highly distorted from its periodicity-free form and may be expected to intersect the less affected slow-cyclotron mode, the position and strength of the intersection being controlled by the geometric properties of the slow-wave structure. Of course, this is also the conventional view of the slow-wave structure interaction with non-relativistic electron beams.[6]

Austin Research Associates, Inc. the inventors of the Auto-Resonant Accelerator principle, have presented a very useful analytic discussion of the slow-wave structure under consideration from the point of view of lumped-element circuit theory.[7] Such an analysis is essential for obtaining an intuitive gra ר of the dynamics of slow cyclotron wave growth by this method. However, this approach makes a number of assumptions whose limits it would be desirable to delineate. Firstly, a treatment strictly from the perspective of Maxwell's equations is desirable in determining those regions of frequency and wave-number where the more tractable lumped-element circuit theory is applicable. Such a determination should provide greater confidence in future analytic studies. Secondly, earlier analysis assumed that the capacitative gaps were so numerous that their capacitance could be assumed to be uniformly distributed in the azimuthal direction. In reality, the capacitance is concentrated at various points about the circumference of the conducting straps, thereby introducing a periodicity in this direction. Such a periodicity can have a marked effect on the relevant mode structure, linking together modes of different azimuthal quantum number. The analysis to be presented below addresses this question. Lastly, previous investigators employed a model beam profile which essentially ignored all radial variations. Numerical investigation has revealed that such a profile is not always appropriate for an electron beam with parameters suitable for collective ion acceleration.[8] An investigation of the effect of a more realistic profile on the growth mechanism is clearly desirable.

The purpose of this paper is to report on some preliminary results of an investigation designed to address these issues. We begin by deriving boundary conditions suitable to the slow-wave structure described above. No recourse is made during this derivation to concepts borrowed from low-frequency circuit theory. These boundary conditions have been incorporated into a numerical code designed earlier to investigate the equilibrium and eigenmodes of a relativistic electron beam propagating in a nonperiodic geometry. This same code may be employed to examine the vacuum modes, an understanding of which is essential to a full appreciation of the cyclotron wave growth mechanism. Finally, we turn to some initial results regarding cyclotron wave growth on a relativistic beam.

The boundary conditions to be applied are simply those that the tangential electric field component $E_\theta(r = a)$ be continuous everywhere and vanish identically on the conducting straps. Further, the radial derivatives are required to be continuous at the gaps. The tangential electric field component $E_z(r = a)$ is, of course, forced to be zero by the presence of the z-slotted waveguide. The usual metallic boundary-value conditions are assumed to be applicable at the outer conducting wall. These conditions may be summarized conveniently as:

$$\sum_{hj}\{E_\theta(r = a)S(\theta - j\theta_0)S(j\theta_0 + \theta_s - \theta)S(z - hL)S(hL + d - z)$$

$$+ \left(\frac{\partial E_\theta^{(1)}}{\partial r}\Big|_{r=a} - \frac{\partial E_\theta^{(2)}}{\partial r}\Big|_{r=a}\right)S\left[\left(j + 1\right)\theta_0 - \theta\right]$$

$$\times S(\theta - j\theta_0 - \theta_s)S(z - hL - d)S(hL + L - z)\} = 0 \qquad (1a)$$

$$E_{\theta,z}^{(2)}\ (r = b) = 0 \qquad (1b)$$

Here L is the axial periodicity length, θ_0 is the corresponding azimuthal quantity, while d and θ_s are respectively the axial length and angular width of the conducting straps. The superscripts (1,2) refer to the regions interior and exterior to the slow wave structure. We have also introduced the standard unit step function:

$$S(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases} \quad .$$

The rather unsightly expression (1a) may be somewhat simplified to yield:

$$\frac{\partial E_\theta^{(1)}}{\partial r} - \frac{\partial E_\theta^{(2)}}{\partial r} + \left[E_\theta^{(1)} - \frac{\partial E_\theta^{(1)}}{\partial r} - \frac{\partial E_\theta^{(2)}}{\partial r} \right] \mathrm{Str}(z,\theta) = 0 \qquad (2)$$

where $\mathrm{Str}(z,\theta)$ is that combination of step functions defining the positions of the straps:

$$\mathrm{Str}(\theta,z) = \sum_{hj} S(z - hL)S(hL + d - z)S(\theta - j\theta_0)$$
$$\times S(j\theta_0 + \theta_s - \theta) \quad . \qquad (3)$$

The numerical code into which the above boundary conditions are to be incorporated solves for the eigenfrequencies and radial eigenfunctions of a mode of the beam-waveguide system which has a specified axial wave number k_z and a particular azimuthal quantum number ℓ. Consequently, Equation (2) must be put into a form consistent with such a scheme. The two-dimensional periodicity of the present slow-wave structure implies that the fields will have the form of Bloch functions:

$$\underline{E}^{(1,2)}(r,z,\theta) = \sum_{\substack{k\ \ell \\ n\ p}} E^{(1,2)}(r)\, e^{i(k+pk_0)z+i(\ell+nm_0)\theta} \quad . \quad (4)$$

The desired form may be obtained by substituting (4) into (2), multiplying by the usual Fourier exponentials, and effecting the necessary elementary integrations. The result is:

$$\begin{aligned}
\bar{E}_\theta^{(lnp)} &+ \frac{1}{\theta_0 L} \sum_{n'p'} \Bigg[\theta_s d - \frac{2\pi L}{m_0}\, \delta_{nn'} \delta_{pp'} \\
&+ \theta_s \frac{\sin(p' - p)k_0 d/2}{(p' - p)k_0/2}\, \delta_{nn'} + \frac{d \sin(n' - n)m_0\theta_s/2}{(n' - n)m_0/2}\, \delta_{pp'} \\
&+ \frac{\sin(n' - n)m_0\theta_s/2 \ \sin(p' - p)k_0 d/2}{(n' - n)m_0 (p' - p)k_0/2} \Bigg] \\
&\times \Bigg[\left(1 + A_{p'}^{n'}\right) \bar{E}_\theta^{(ln'p')}(r = a) \\
&- \frac{\partial \bar{E}_\theta^{(ln'p')}(r)}{\partial r} \Bigg|_{r=a} \Bigg] = 0 \qquad (5)
\end{aligned}$$

where we have defined:

$$\bar{E}_\theta^{(lnp)}(r = a) = e^{i(pk_0 d/2 + nm_0\theta_s/2)}\, F_\theta^{(lnp)}(r = a) \qquad (6)$$

and

$$A_p^n = \Big[\Big(I'_{\ell+nm_0+1}\,(k_p a) + I'_{\ell+nm_0-1}\,(k_p a)\Big)$$

$$\times \Big(K_{\ell+nm_0+1}\,(k_p b) + K_{\ell+nm_0-1}\,(k_p b)\Big)$$

$$- \Big(I_{\ell+nm_0+1}\,(k_p b) + I_{\ell+nm_0-1}\,(k_p b)\Big)$$

$$\times \Big(K'_{\ell+nm_0+1}\,(k_p a) + K'_{\ell+nm_0-1}\,(k_p a)\Big)\Big]\Big/D \quad . \qquad (7a)$$

$$D = [I_{\ell+nm_0+1}\,(k_p a) + I_{\ell+nm_0-1}\,(k_p a)]$$

$$\times [K_{\ell+nm_0+1}\,(k_p b) + K_{\ell+nm_0-1}\,(k_p b)] - [a \leftrightarrow b] \quad . \qquad (7b)$$

$$k_p^2 = \frac{\varepsilon\omega^2}{c^2} - \left(k_z + \frac{2\pi p}{L}\right)^2 \quad . \qquad (7c)$$

The expression (7) is derived from a consistent application of boundary condition (1b) together with that deriving from the continuity of the tangential electric field. Here $k_0 = 2\pi/L$ and m_0 is the analogous, but integral, quantity, which is in fact the number of azimuthal gaps employed. The prime on the summation symbol denotes, as usual, the omission of terms which would give rise to singularities through the vanishing of denominators. The quantity ε is the dielectric constant of the material which fills the region between the slow wave structure and the outer conducting wall. This material has been introduced for numerical tuning, as will be elaborated further below.

Equation (5) could perhaps be made the basis of analytic study of slow cyclotron wave growth. Such a study would require several assumptions and approximations whose validity in parameter regimes of experimental interest is not always clear. Our immediate objective has been rather to employ Equation (5) to conduct numerical investigations which are not limited by such assumptions and approximations. Previsouly,[8] a numerical code, GRADR, was written, which constructs self-consistent beam equilibria and performs a normal mode analyses of linear perturbations made about such equilibria. The code produces both the eigenfrequency and the corresponding radial eigenfunctions of a given mode. The equilibria examined have a number of features not shared by the model equilibria generally employed in analytic studies. In particular, the radial variation of the relativistic factor induced by the presence of the space charge is automatically included. This variation has a profound effect, both on the form of the radial eigenfunctions and on the overall appearance of the dispersion diagram. While the dispersion properties of the slow cyclotron wave under discussion are but little modified, the radial eigenfunctions are considerably modified from the Bessel function form characteristic of uniform radial profiles. Particularly striking is the peaking of the relevant eigenmode about the edge of the beams. In addition, discrete modes which appear in the uniform theory are replaced by bands of continuous modes. These features can have significant consequences for cyclotron wave growth mechanisms, leading, for example, to the necessity of greater radial modulation than that predicted by the uniform theory and to the shifting of relevant discrete modes into the regions of continua. These questions have been extensively investigated for growth by helical slow wave structures,[5] but are yet to be addressed for the loop-drive.

The numerical code described above has been modified to include the periodic boundary conditions displayed in Eq. (5).

The code may also be used to examine the purely vacuum modes of the slow wave structure. Such an examination is necessary for a full understanding of the interaction when a beam is present. Study of the vacuum modes is also useful in assessing the accuracy of various truncations which must be effected when using Eq. (5). We expect that the vacuum mode will be relatively flat and that it will exhibit a periodicity in w-k space given by k_0. The degree to which this periodicity is observed may be taken as a measure of the accuracy of a given truncation scheme. These expectations have been fully realized. Runs with $k_0 = 1$ and $d/L = 0.5$-0.9 have revealed modes which vary in frequency by approximately 15 percent throughout a Brillouin zone. With three axial zones the frequency was periodic to within 2 percent, while with 5 it was periodic to within a tenth of a percent. These results were but little changed when azimuthal periodicity was included.

A further question which may be addressed through a study of the vacuum modes is that of tuning the slow wave structure. One would like to achieve the growth of waves with phase velocities roughly in the range of 0.1-0.25. For parameters typical of Auto-Resonant Accelerator operation, this corresponds to a resonant frequency for the slow wave structure of $w_0 = 0.06$-$0.1\ \omega_p$. This frequency is, of course, a function of the various geometrical factors involved, and one might believe that a judicious choice of these quantities would lead to the desired value. Actually, it was found difficult to reduce this frequency much below 0.2 without losing significant coupling between the various components. This difficulty can probably be traced to our idealization of the gaps as having no radial extent. The inclusion of a finite radial width would presumably lead to a greater effective capacitance in the equivalent circuit of the slow wave structure and consequently a lower resonant frequency. In the analysis of such a system one must recognize that the azimuthal and axial dependencies of the fields within the gaps are not identical to those occurring in the interior and exterior regions. Consequently, several important simplifications which occurred in the derivation of (5) do not appear, and the calculation rapidly becomes unwieldy. Rather than pursue this course, we have instead resorted to the simple expedient of filling the region between the slow wave structure and the exterior wall with a substance of constant dielectric constant $\varepsilon \cong 30\text{-}60$. This quantity can now be adjusted to yield the desired resonant frequency. It is to be stressed that although the dielectric is being introduced here purely to achieve the desired tuning, it is not altogether clear that the presence of such a substance is not possible or desirable in the actual system. This issue must await the resolution of dielectric breakdown questions. Nevertheless, using this procedure with dielectric constants in the range 30-60, we have been able to produce vacuum modes of the desired frequency. Cyclotron wave growth at the desired phase velocity has not, however, yet been achieved.

Preliminary results for cyclotron wave growth at a somewhat higher phase velocity are exhibited in Fig. 2.

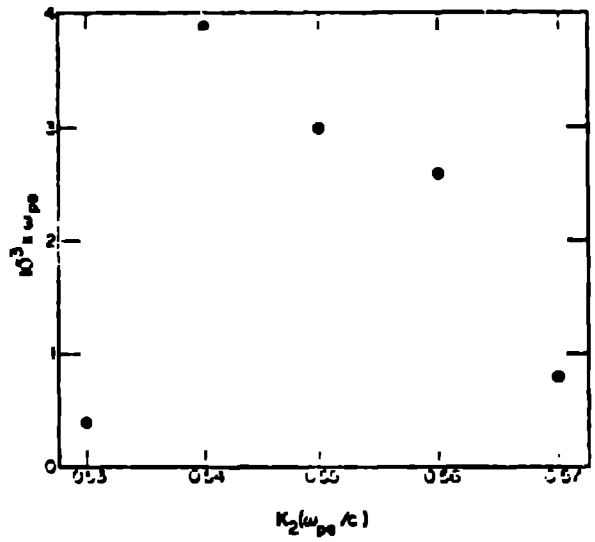

Fig. 2. Cyclotron wave growth with $\gamma = 7$, $R_{beam} = 2.65\ c/\omega_p$, $a = 3.8\ c/\omega_p$, $b/a = 10$, $\Omega_c = 2.0\ \omega_p$.

Although the parameters chosen to generate this graph do not necessarily optimize the growth rate, examination of the results of this run reveal a number of features which are likely to persist under more favorable circumstances. Firstly, as is clear from the graph itself, the region of growth is very narrow in w-k space. This is in marked contrast to the case of the helical structure, which is a broad-band amplifier. Such sharpness of the resonance may prove an important advantage from the point of view of coherence, provided that it does not seriously militate against initial excitation of the desired mode. Further examination reveals significant coupling between the principal mode and those lying immediately adjacent diffraction zones, the ratio of amplitudes being roughly 0.25. Coupling to more distant zones is much less. Some concern may therefore arise that unwanted modes will experience significant growth. Actually, such concern is unwarranted in the present case, since the relevant modes lie in bands of the continuous modes referred to above. Previous investigation has revealed that such modes, if excited, tend to phase mix away in a secular fashion.

Further study along the lines sketched here is clearly required to ascertain whether this slow wave structure will provide an effective growth of the slow cyclotron waves for Auto-Resonant Acceleration. The linear theory code described above will be used in the near future to determine those beam and structure parameters which lead to optimal phase velocity and growth rate. The important question of the growth of modes with higher principal azimuthal quantum numbers will also be addressed. All the information thereby gained will be used to choose parameters with which to perform cylindrical, relativistic, fully electromagnetic particle computer simulations[9] of slow cyclotron wave growth by the resonant loop-drive.

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